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A METHODOLOGY TO ESTIMATE THE VOLUME AND STEM TAPER BASED ON SOLIDS OF REVOLUTIONS

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ABSTRACT

The knowledge and identification of stem forms allow more accurate quantification of timber resources as the volume and knowledge of aspects related to species genetics and environmental factors on tree growth. Therefore, the objective of this study was the development of methodology based on concepts of solids of revolution to identify the different forms along the tree stems; define the most appropriate formulas to compute the volume of each part of the stem; and estimate diameters along the stems (stem taper). For this, data from 27 felled sample trees of *Eucalyptus grandis*, separated equally into nine diameter classes, with 60 months of age, planted under 3 × 3 meters spacing were used to establish the relationships between the variables and develop the methodology. For the set of sample trees, the basal part of the stems resembled a neiloid, a cone at the center, and a paraboloid at the top. The differences between stem volumes obtained by applying specific formulas for each stem section and those obtained by a single formula for all sections were statistically significant (*p-value* < 0.05). The proposed methodology provided accurate estimates of diameters outside the bark compared to the diameters estimated by the taper models for most sample trees, showing the potential of using the methodology as an alternative to traditional methods.

Keywords: Geometric forms; *Eucalyptus* trees; Taper functions.

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UMA METODOLOGIA PARA ESTIMAR O VOLUME E O AFILAMENTO DO TRONCO BASEADA EM SÓLIDOS DE REVOLUÇÃO

RESUMO – O conhecimento e a identificação das formas do tronco permitem uma quantificação mais precisa dos recursos madeireiros quanto ao volume e o conhecimento de aspectos relacionados à genética das espécies e aos fatores ambientais no crescimento das árvores. Assim sendo, o objetivo deste estudo foi o desenvolvimento de metodologia baseada em conceitos de sólidos de revolução para identificar as diferentes formas ao longo dos troncos das árvores; definir as fórmulas mais adequadas para calcular o volume de cada parte deles; e estimar os diâmetros ao longo dos troncos (afilamento). Para isso, foram utilizados dados de 27 árvores amostrais de *Eucalyptus grandis*, separadas igualmente em nove classes de diâmetro, com 60 meses de idade, plantadas no espaçamento 3 × 3 metros, para estabelecer as relações entre as variáveis e desenvolver a metodologia. Para o conjunto das árvores amostrais, verificou-se que a parte basal dos troncos assemelhavam-se a um neilóide; a um cone na parte central; e a um parabolóide no topo. As diferenças entre os volumes do tronco obtidos pela aplicação de fórmulas específicas para cada seção do caule e aqueles obtidos por uma fórmula única para todas as seções foram estatisticamente significativos (*p-valor* < 0,05). A metodologia proposta forneceu estimativas precisas dos diâmetros com casca em comparação com os diâmetros estimados pelos modelos de afilamento para a maioria das árvores amostradas, mostrando o potencial do uso da metodologia como alternativa aos métodos tradicionais.

Palavras-Chave: Formas geométricas; Árvores de *Eucalyptus*; Funções de afilamento.

1. INTRODUCTION

The planting of commercial species generally exhibits an excurrent crown shape and has a single main stem (Burkhart and Tomé, 2012), which is used to produce solid wood products, cellulose pulp, charcoal, and others. Parts of the stem of these trees resemble different geometric solids, and factors, such as competition between trees, social position inside the forest, silvicultural treatments, spacing of planting, age, and size of the trees influence stem form (Van Laar and Akça, 2007; Ferreira et al., 2014; Kohler et al., 2016; Cerqueira et al., 2021), increasing the difficulty of establishing a pattern where multiple inflection points occur along the stems (Burkhart and Tomé, 2012).

Although there are many methods to estimate the volume and the stem taper (Husch et al., 2003; Weiskittel et al., 2011; Burkhart and Tomé, 2012; Andrade, 2014; McTague et al., 2020), identifying the different geometric forms along the stems of trees to make it possible to estimate diameters and volumes more accurately considering each one of them remains a problem. Because of this difficulty, the stems are divided into logs (sections) of determined lengths, and the volumes are computed considering a specific formula for all sections (Figueiredo Filho et al., 2000, Campos and Leite, 2017). Therefore, the diameters and volumes obtained by this procedure are used to fit the volume and taper equations to estimate tree and stand volumes (Husch et al., 2003).

A volume equation allows the estimation of a single volume, whereas taper equations can estimate multiple volumes (Weiskittel et al., 2011) by different portions of the stem. For this reason, a variety of taper models exist (Rojo et al., 2005; Andrade, 2014), and some categories as segmented (Max and Burkhart 1976) and variable-form (or exponent) functions (Kozak, 1988; Newnham, 1988) consider different forms along the stems to estimate the volumes, showing the importance of the form as an attribute to increase the accuracy of the models.

Given the importance of identifying the different forms along the stems for forest mensuration and management, silviculture, and the ecology of tree species, we conducted this study using methodological development based on the solids of revolution, as an alternative to traditional methods of development, to identify the different forms along the *Eucalyptus* tree stems, define the most appropriate formulas to compute the volume of each part of the stem, and allow the estimation of diameters along the stems with greater precision compared to traditional methodologies.

2. MATERIAL AND METHODS

2.1 Study area

The study was conducted in Viçosa, Minas Gerais State, Brazil (20°45′S, 42°52′W; 648 m a.s.l.). According to the Köppen-Geiger classification, the local climate is Cwa. The annual average temperature is 21.9 °C, and the annual precipitation is 1,274 mm (UFV, 2016). The topography is mountainous, with steep slopes and narrow, humid valleys. Red-yellow alsicose latosol predominates at the top of hills and slopes and the cambic yellow-red podzolic in terraces (Ferreira Júnior et al., 2009).

2.2. Data collection

Data from 27 felled sample trees of *Eucalyptus grandis*, selected selectively and divided equally into 9 diameter classes, with 60 months of age, planted under 3 × 3 meters spacing and including different sizes, were used for methodological development. In each sample tree, were measured the diameters outside the bark (d) along the stem to each 0.5 m up to a minimum or limit diameter (dl), defining the commercial height (Hc), diameter at breast height (dbh) and total height (H) (Table 1).

Table 1 - Range of the sample trees data use to develop the proposed methodology (diameter at breast height (*dbh*); total height (*H*); commercial height (*Hc*); minimum ou limit diameter along the stems (*dl*), defining *Hc*

Tabela 1 - Faixa de dados das árvores amostrais utilizadas para desenvolver a metodologia proposta (diâmetro à altura do peito (*DAP*); altura total (*H*); altura comercial (*Hc*); diâmetro mínimo ou limite ao longo do tronco (*dl*), definindo *Hc*

Variable	Minimum	Maximum
dbh (cm)	15.75	25.40
H (m)	24.30	31.80
Hc (m)	20.00	30.50
dl (cm)	2.03	5.84

2.3 Generating solids of revolution

The solids of revolution can be generated according to the following equations (Husch et al., 2003; Campos and Leite, 2017):

$$Y = k \times \sqrt{h^b} \quad (Eq.1)$$

$$Y = k \times h^r \quad (Eq.2)$$

Here, Y is the radius of the cross-sections along the stems, k is a constant; h is the height at which Y occurs, and $r = b/2$, is the coefficient of the form that defines a specific solid of revolution (if $r = 0$, define a cylinder; $r = 0.5$, a paraboloid; $r = 1$, a cone; and $r = 1.5$, a neiloid).

In this study, we modified Equation 2 to generate the solids of revolution (cylinder, paraboloid, cone and neyloid) for each sample tree in an inverted way, that is, rotating the curve around the Y axis, such that:

$$Y = \frac{R_b}{H^r} \times (H - h)^r \quad (Eq.3)$$

where Y is the radius of the cross-sections along the stems, R_b is the radius of the cross-section at the base of the stem (equal diameter at the base (d_b) divided by 2), h is the height at which Y occurs, H is the total height and r = coefficient of the form. So, when $h = 0$, $Y = R_b$ and $h = H$, $Y = 0$.

2.4 Associating observed diameters to the solids of revolution

After generating the solids of revolution using Equation 3, the observed diameters outside the bark along the stem of each tree were plotted in graphics to show the distribution among the solids. As the observed diameters were not exactly over them, the following equation, defined by isolating the coefficient of the form (r) in Equation 3, was used to make it possible to associate a solid of revolution with each diameter along the stem (each 0.5m):

$$r = \frac{ln(R_b) - ln(Y)}{ln(H) - ln(H - h)} \quad (Eq.4)$$

where ln is the neperian logarithm.

Thus, the coefficients (r) along the stems

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could be classified considering the following intervals, associated with the solids of revolution along the tree stems: if $0 \leq r < 0.25$, a cylinder; $0.25 \leq r < 0.75$, a paraboloid; $0.75 \leq r < 1.25$, a cone; and $r \geq 1.25$, a neiloid.

2.5 Volume determination

Once the forms along the stems (cylinder, neiloid, cone, or paraboloid) and their transitions were obtained, the volume outside the bark of each section (considering lengths of 0.5 m) was calculated by applying the formulas considering the frustum of the cone, paraboloid, and neiloid, respectively (Husch et al., 2003):

$$V = \frac{1}{3}\left(g_2 + (g_2 g_1)^{\frac{1}{2}} + g_1\right) \times L \qquad (Eq.5)$$

$$V = \left(\frac{g_2 + g_1}{2}\right) L \qquad (Eq.6)$$

$$V = \frac{1}{4} \times \left(g_2 + (g_2^2 g_1)^{\frac{1}{3}} + (g_2 g_1^2)^{\frac{1}{3}} + g_1\right) \times L \qquad (Eq.7)$$

where g_2 and g_1 are the circular areas at the end and beginning of the sections, respectively, and L is the section length (0.5m).

The total volumes outside the bark of the tree stems, obtained by applying Equation 5, 6 or 7 for each section separately, according to their respective forms (proposed methodology), were compared to the estimates considering the stems having only one form, that is, obtained by applying Equation 5, 6 or 7 to all sections of the stem (Vcone, Vpar, Vnei).

For this comparison, Graybill's F-test (Graybill, 1976) was carried out to verify if the volumes obtained by the proposed methodology are statistically different of the other volumes estimates, considering a linear model and evaluating simultaneously if the parameters β_0 and β_1 were statistically equal to 0 and 1 (Nascimento et al., 2020).

2.6 Stem taper: A new methodology

The proposed methodology based on the observed diameters, described in subsections 2.3, 2.4, and 2.5, allows the form (r) calculation, identifies the forms along the tree stems, and determines their volumes, respecting these forms. However, it can also be used to reconstruct the longitudinal profiles of tree stems (stem taper).

By analyzing Equation 3, it is observed that the following variables are necessary to estimate the radius (Y) or diameters (dcc) along the tree stems and, consequently, reconstruct the longitudinal profile (stem taper): radius or diameter at the base of the tree (Rb/db); total height (H); heights where the diameters (h) and the coefficients of the form (r) occur along the stem. Thus, regression models were evaluated only to estimate the dependent variables Rb/db, H and r, because h values can be simulated along the stem to allow reconstruction of the longitudinal profile of the stems (stem taper).

The modelling was initially performed based on correlations between variables and graphical analysis of variable distributions. The selection of the best-fitted equation for each dependent variable was based on the coefficient of determination (R^2) and standard error of estimate ($S_{y.x}$) (Draper and Smith, 1998), obtained using the software R (R Core Team, 2019).

2.7 Taper models

Taper equations according to the models of Kozak et al. (1969), Demaerschalk (1972), Ormerod (1973), and Kozak (1988) (Equations 8, 9, 10, and 11, respectively) were fitted to observed data from sample trees to make it possible to compare the results of this traditional methodology with the proposed methodology based on solids of revolution (subsection 2.6):

$$\left(\frac{dcc}{dbh}\right)^2 = \beta_0 + \beta_1 \times \left(\frac{h}{H}\right) + \beta_2 \times \left(\frac{h}{H}\right)^2 + \varepsilon \qquad (Eq.8)$$

$$\left(\frac{dcc}{dbh}\right)^2 = 10^{2\beta_0} \times dbh^{2\beta_1 - 2} \times (H-h)^{2\beta_2} \times H^{2\beta_3} + \varepsilon \qquad (Eq.9)$$

$$\left(\frac{dcc}{dbh}\right)^2 = \left(\frac{H-h}{H-1.3}\right)^{2\beta_1} + \varepsilon \qquad (Eq.10)$$

$$\begin{aligned} ln(dcc) = {} & \ln(\alpha_0) + \alpha_1 \times ln(dbh) + ln(\alpha_2) \times dbh \qquad (Eq.11) \\ & + \beta_1 \times ln(z) x^2 \\ & + \beta_2 \times ln(z) \times ln(x + 0.001) \\ & + \beta_3 \times ln(z) \times \sqrt{x} \\ & + \beta_4 \times ln(z) \times e^x \\ & + \beta_5 \times ln(z) \times \left(\frac{dbh}{H}\right) + \varepsilon \end{aligned}$$

where: $z = \frac{1 - \sqrt{h/H}}{1 - \sqrt{p}}$; $x = h/H$

dcc is diameters outside bark along the stems, in centimeters (cm); dbh is diameter outside bark at 1.30m, in cm; h is the height where a given diameter (dcc) occurs, in meters; H = total height, in meters; α_0, α_1, α_2, β_0, β_1, β_2, β_3, β_4, and β_5 = model`s parameters; ε = random error; and p is a fix proportion of the total height where the stem changes form.

The parameter estimates were obtained using R software (R Core Team, 2019). For the non-linear models (Equations 9 and 10) was used the nls function (Gauss-Newton algorithm). The criteria used to verify the precision of the fitted equation were the empirical adjusted R-squared ($\bar{R}^2$) and standard error of the estimate ($S_{y.x}$) (Crescente-Campo et al., 2010). The significance of the parameters was verified by the t-test and *p-value*. The variance inflation factor (VIF) assessed multicollinearity, with values higher than 10 indicating collinearity (O`Brien, 2007; Ribeiro et al., 2014).

2.8 Comparing taper methodologies

The accuracy of the diameter estimates from the proposed methodology based on solids of revolution and taper equations ($\hat{y}_i$) in relation to the diameters outside bark measured along the stems of the sample trees (*yi*) was verified by graphical analysis and by calculating the root mean square error (RMSE) and mean absolute deviation (MDA) (Souza et al., 2018):

$$RMSE = \sqrt{\frac{\sum(y_i - \hat{y}_i)^2}{n}} \qquad (Eq.12)$$

$$MAD = \frac{\sum|y_i - \hat{y}_i|}{n} \qquad (Eq.13)$$

where *n* is the number of observations.

3. RESULTS

3.1 Forms along of the stems

As an example, applying Equation 3 for one sample tree (tree # 2), whose *dbh* is 16.76 cm, total height (*H*) and commercial (*Hc*) equal to 27.7 m and 26.0 m, respectively, and radius at the base of the stem (R_b) equals 9.15 cm, the solids of revolution were generated (Figure 1), where it is observed the diameters outside bark between the forms along the stems.

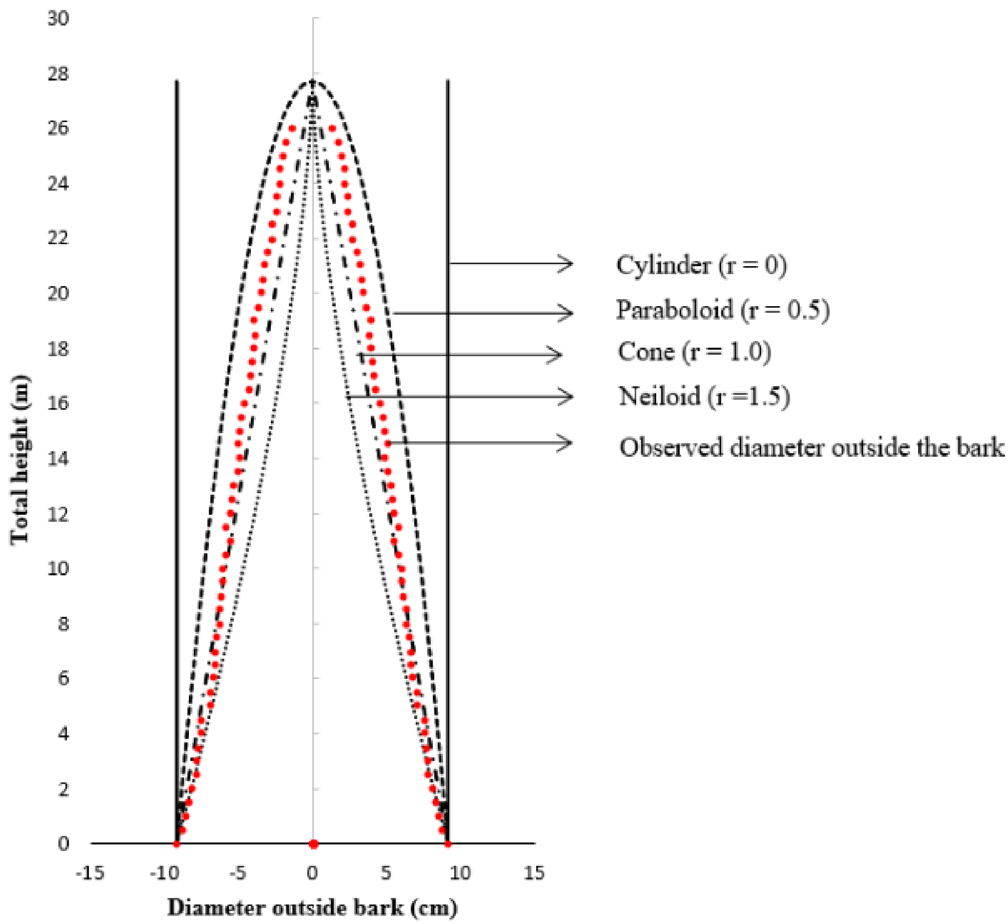

Figure 1 - Solids of revolutions generated for sample tree #2 and observed diameters outside bark along the stem.
Figura 1 - Sólidos de revoluções gerados para a árvore de amostra 2 e diâmetros observados com casca ao longo do tronco.

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The coefficients of the form (r) along the stem of tree #2, applying Equation 4, ranged from 0.66 to 2.31, decreasing from the base to the top of the tree. Classifying the coefficients of the form (r) according to the intervals defined in the methodology (subsection 2.4), the stem of sample tree #2 resembles a neiloid if height (h) is less than 4.0 meters; a cone if it is $4.0 \leq h < 23.0$ m, and a paraboloid if it is $23.0 \leq h \leq 26.0$ m.

The trend remained the same for the set of sampled trees, with the basal part of the tree stems resembling a neiloid, a cone at the central portion, and a paraboloid at the top. However, they occur at different heights (Figure 2), even for trees of similar size.

3.2 Volume determination

The differences between the total volume estimates for the set of the sample tree (Figure 3), applying Equation 5, 6 and 7 for each section separately (Vsolids), according the porposed methodology, and the estimates considering the stems with only one form (Vcone, Vpar, Vnei) were minor (maximum equal 0.04%). However, the volumes calculated by the proposed methodology (Vsolids) were statistically differents of the other volumes estimates (Vcone, Vpar, Vnei), considering the results of Graybill's F-test (*p-value* < 0.05), owing to the tendency of under and overestimation when applying a single equation to estimate volumes along the stems.

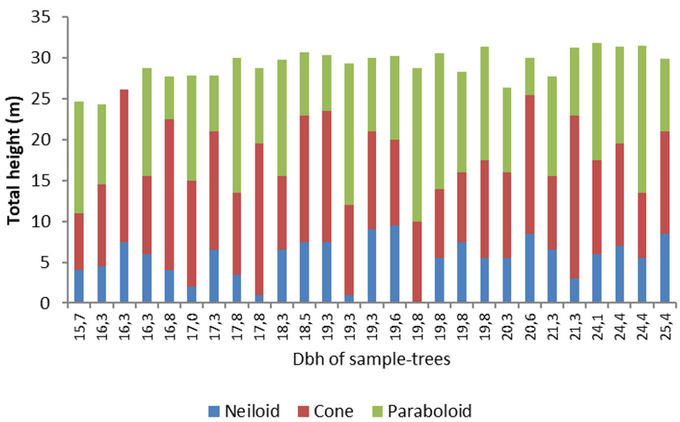

Figure 2 - Occurrence of different forms (neiloid, cone, and paraboloid) along the stems of sample trees.
Figura 2 - Ocorrência de diferentes formas (neilóide, cone e parabolóide) ao longo dos troncos das árvores amostradas.

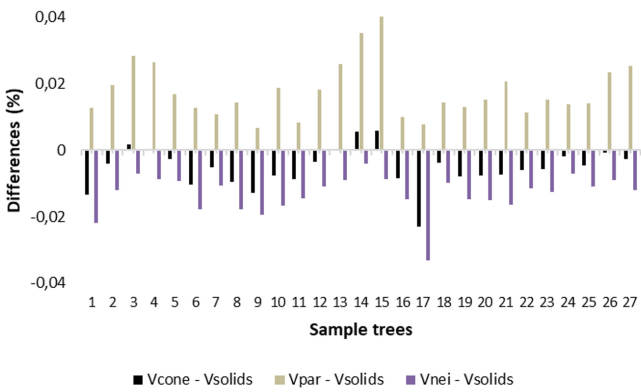

Figure 3 - Differences in percentage between the volumes of the sample trees obtained by the frustums of the cone (Vcone), paraboloid (Vpar), and neiloid (Vnei) formulas applied to all sections of the stems and for each section separately according to the classification of forms (Vsolids).
Figura 3 - Diferenças percentuais entre os volumes das árvores amostrais obtidas pelas formulas dos troncos de cone (Vcone), parabolóide (Vpar) e neilóide (Vnei), aplicadas a todas as seções dos tronco e para cada seção separadamente de acordo com o classificação das formas (Vsolids).

The stems of the trees resemble a neiloid in their basal part (Figure 2), with a shorter length among the different solids (19.0% of the total height of the trees), but they represent an average of 39.8% of the volume (Table 2). The central part of the trees resembles a cone, with greater length (42.6%) and more significant contribution in volume (46.4%). The final part of the tree stem, which resembles a paraboloid, represents an average 38.4% of the total heights of the trees but contributes 13.8% of the volume only.

Table 2 - Average lengths of the stem sections ($\bar{L}$) and representation in terms of the average percentage of total tree height ($\overline{\%H}$) and volume ($\overline{\%V}$)* by diameter classes and geometric forms

Tabela 2 - Comprimentos médios das seções dos troncos ($\bar{L}$) e representação em termos da porcentagem média da altura total da árvore ($\overline{\%H}$) e do volume ($\overline{\%V}$)* por classes de diâmetro e formas geométricas

Diameter classes (cm)	$\bar{H}$ (m)	Neiloid			Cone			Paraboloid		
		$\bar{L}$ (m)	$\overline{\%H}$	$\overline{\%V}$	$\bar{L}$ (m)	$\overline{\%H}$	$\overline{\%V}$	$\bar{L}$ (m)	$\overline{\%H}$	$\overline{\%V}$
15.0 – 17.49	26.7	4.9	18.5	34.0	13.0	48.5	52.5	10.3	33.0	13.5
17.5 – 19.99	29.8	5.8	17.8	34.3	11,8	39.5	46.5	12.7	42.7	19.2
20.0 – 22.49	28.8	5.9	20.6	37.9	14.1	48.3	50.8	8.8	31.2	11.3
22.5 – 24.99	31.6	6.2	19.5	38.9	10.7	33.8	42.6	14.7	46.7	18,5
25.0 – 27.49	29.9	8.5	28.4	53.7	12.5	41.8	39.8	8.9	29.8	6.5
Average		5.7	19.0	39.8	12.4	42.6	46.4	11.6	38.4	13.8

*Volumes calculated by the proposed methodology

3.3 Taper equations

The equations related to Kozak et al. (1969), Demaerschalk (1972), and Ormerod (1973) models fitted well with the observed data ($\bar{R}^2$ > 0.94) and all parameters were statistically significant (*p-value* ≤ 0.05) (t-test values between parenthesis):

(Eq.14)

$$\left(\frac{dcc}{dbh}\right)^2 = 1.052485 - 1.673238 \times \left(\frac{h}{H}\right) + 0.659619 \times \left(\frac{h}{H}\right)^2$$

(232.39) (-75. 21) (28.82)

$\bar{R}^2 = 0.9591$; $S_{y.x} = \pm 0.06075$cm

(Eq.15)

$$\left(\frac{dcc}{dbh}\right)^2 = 10^{-0.61114} \times dbh^{0.134054} \times (H-h)^{1.384254} \times H^{-1.078298}$$

(-9.16) (72.99) (125.87) (-18.33)

$\bar{R}^2 = 0.9576$; $S_{y.x} = \pm 0.06183$cm

(Eq.16)

$$\left(\frac{dcc}{dbh}\right)^2 = \left(\frac{H-h}{H-1.3}\right)^{1.464748}$$

(152.4)

$\bar{R}^2 = 0.9494$; $S_{y.x} = \pm 0.06756$cm

Some coefficients of the original (complete) Kozak`s model (Equation 11) were statistically non-significant (*p-value* > 0.05) because of multicollinearity. Variables with a variance inflation factor (VIF) greater than 10 were excluded from the analysis, and only significant variables (i.e., *p-value* < 0.05) were retained in the final models. The fitted variable-exponent taper equation at the end of this modelling process is (t-test values between parenthesis):

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$$ln(dcc) = -1.84166 + 1.781777 \times ln(dbh) \quad (Eq.17)$$
$$\quad (-4.40) \qquad (8.67)$$
$$-0.03251 \times dbh - 0.46273 \times ln(z) \times x^2$$
$$\quad (-3.19) \qquad (-28.31)$$
$$+0.364322 \times ln(z) \times e^x$$
$$(68.41)$$
$$\bar{R}^2 = 0.9740; \quad S_{y.x} = \pm 0.0747 \text{ cm}$$

The proportion of the total height or relative height (p) used to calculate the variable z (Equation 11), which defines the inflection point where the stem changes in form, was 0.19 (% H), for the transition from neiloid to cone (Table 2).

Applying Equations 14, 15, 16, and 17 to sample tree #2, the diameters outside the bark to each 0.5 m along the stem up to a height (h) equal to 26.0 m were estimated (Figure 4A, 4B, 4C, and 4D), describing the stem taper of this sample tree by the different taper models.

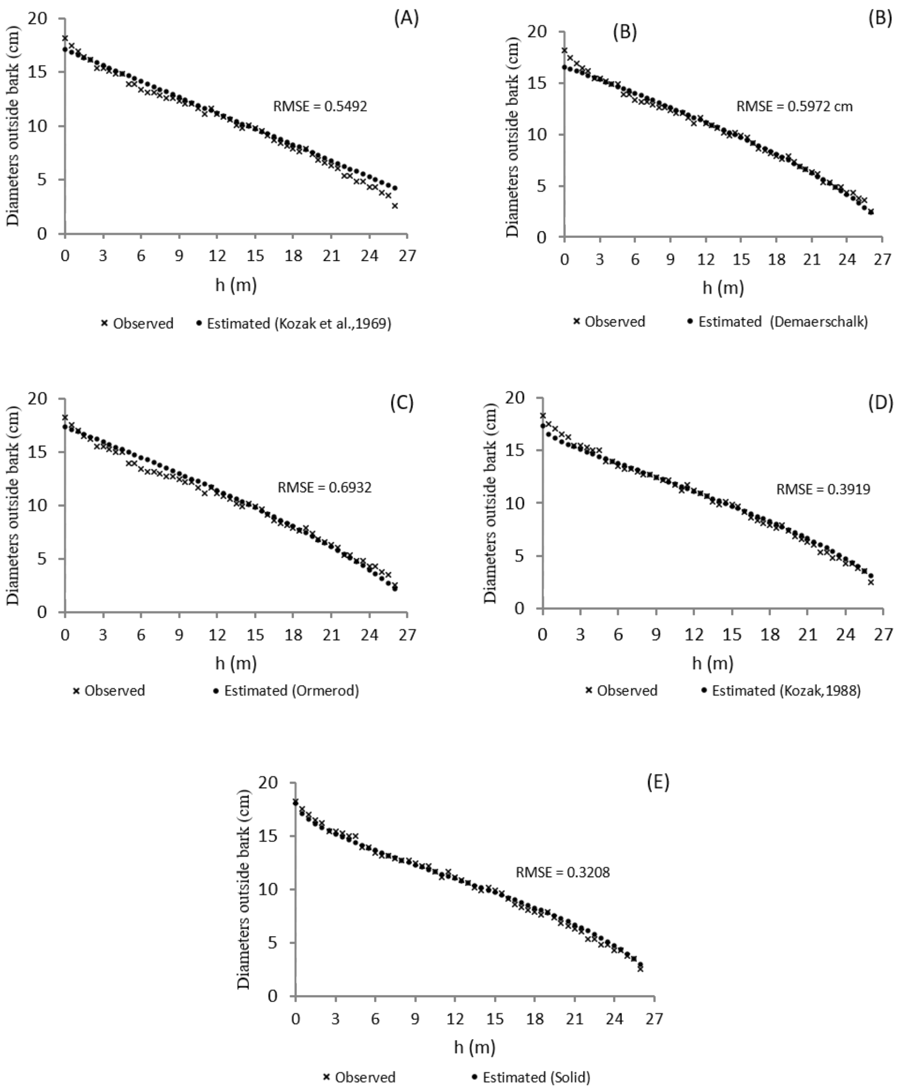

Figure 4 - Distribution of the observed diameters outside bark and estimated by the taper models (A, B, C, and D) and by the proposed methodology (E) along the stem of the sample tree #2.

Figura 4 - Distribuição dos diâmetros com casca observados e estimados pelos modelos de afilamento (A, B, C e D) e pela metodologia proposta (E) ao longo do tronco da árvore amostra 2.

The fitted taper equations presented inaccurate estimates for the diameters outside bark at the bottom of the stem. The Kozak et al. (1969) model also provided inaccurate estimates of the diameters at the stem top.

3.4 Taper by solids of revolutions

The curve of the form (r) coefficients about the relative heights (h/H) where they occur, considering all sample trees, follows a decreasing trend. In contrast, the height-diameter ratio, an increasing curve, tends to an asymptotic value, and the curve of the diameters at the base of the stems (db) whith the *dbh* of the trees, which is a linear relationship (Figures 5A, 5B, and 5C).

After the modeling process, the tendencies of the curves showed in Figures 5A, 5B, and 5C can be expressed by the following equations (t-test values between parenthesis):

$$LnH = 3.745406 - 7.24446 \times \left(\frac{1}{dbh}\right) \quad (Eq.18)$$

(81.17) (-81.57)

R^2=0.7398; $S_{y.x}$=± 0.1742

$$d_b = -3.5543 + 1.2899 \times dbh \quad (Eq.19)$$

(45.76) (-4.65)

R^2=0.4641; $S_{y.x}$=± 0.0536ln(m)

$$r = 0.6251 \times \left(\frac{h}{H}\right)^{-0.394} \quad (Eq.20)$$

(-3.09) (22.08)

R^2=0.9512; $S_{y.x}$=± 0.7955cm

3.4.1 Case of study

As an example of the application of the developed methodology by using Equation 18, 19, and 20 to reconstruct the profile of the stems, substituting the dbh of sample tree #2 (16.76 cm) initially in Equation 19 and 20 were estimated the total height (H) of the tree (27.50 m), as well as the diameter at the base of the stem (db) (18.10 cm) and, consequently, the radius at the base (Rb) (9.05 cm), which were very close to the observed values (H = 27.70 m and db = 18.30 cm, respectively).

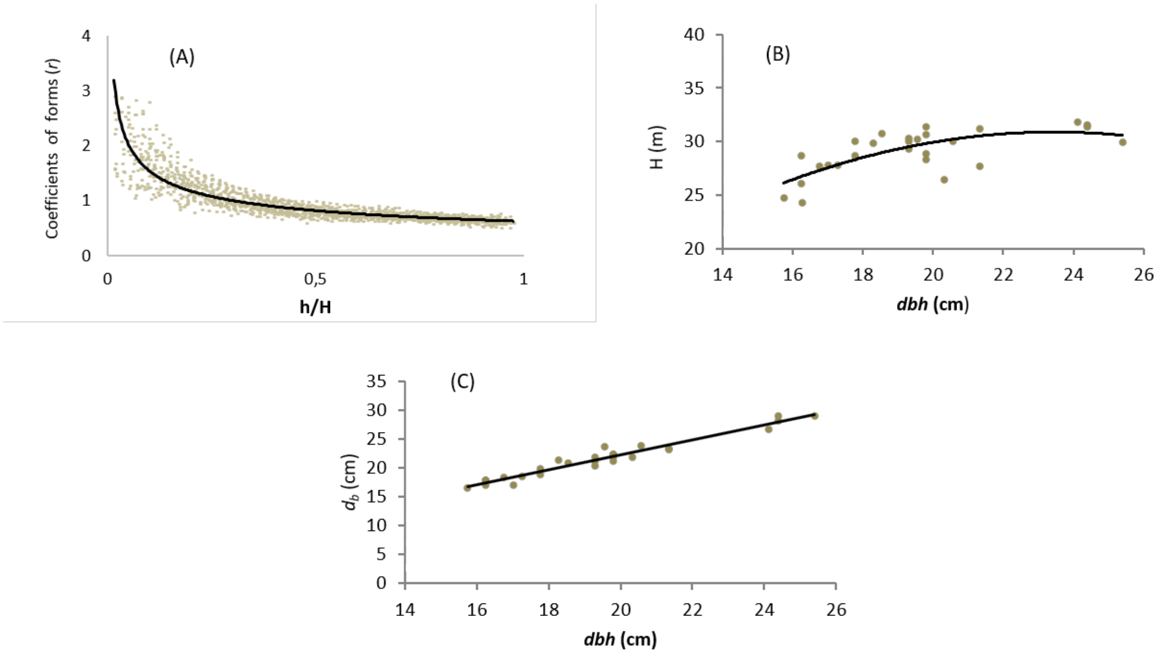

Figure 5 - Trend curves between coefficients of forms (r) and relative heights (h/H) (A); height-diameter relationship (B), and diameter at the base of the stems (db) and *dbh* (C).

Figura 5 - Curvas de tendência entre coeficientes de formas (r) e alturas relativas (h/H) (A); relação altura-diâmetro (B), e diâmetro na base dos troncos (db) and *DAP* (C).

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Substituting the estimate of total height (H) and height values (h) in Equation 18, in intervals of 0.5 m up to the height equal to 26.0 m, were obtained estimates of the coefficients of the form (r) along the stem, which ranged from 3.04 (base) to 0.64 (top).

By the classification of the estimates of coefficients (r) in the intervals defined in subsection 2.4 of the methodology, the stem of sample tree #2 resembles a neiloid if height (h) less than 4.5 m; a cone if $4.5 \leq h < 17.0$ m, and a paraboloid if $17.0 \leq h \leq 26.0$ m, a result slightly different from the values presented at the beginning of the results.

Substituting the estimates of coefficients (r) along the stem in Equation 3, varying h up to a height equal to 26.0m, and considering the estimates of $H = 27.50$ m and $R_b = 9.05$, it was possible to estimate the radius of the cross-sections along the stem and, consequently, the respective diameters outside bark for sample tree #2 (Figure 4E).

3.5 Comparison between methodologies

Compared to the taper equations (Equation 14, 15, 16, and 17), the proposed methodology used to reconstruct the longitudinal stem profile (stem taper) of sample tree #2, based on solids of Revolution; was more accurate (RMSE = 0.3208 cm), considering the entire stem and providing accurate diameter estimates at the base of the stem (Figure 4E). For the set of sample trees, the proposed methodology (based on solids of revolution) resulted in lower estimates of MAD and RMSE for

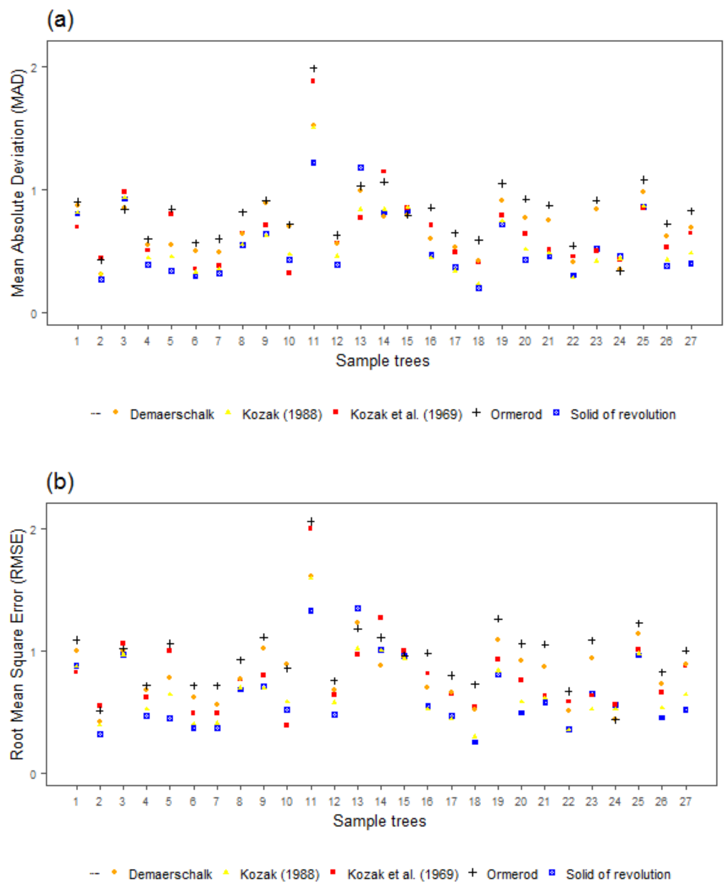

Figure 6 - Mean Absolute Deviation (MAD) (A) and Root Mean Square Error (RMSE) (B) for the sample trees calculated considering diameter estimates by the proposed methodology (Solids of revolution) and the taper models (Demaerschalk; Kozak 1988; Kozak et al. 1969; and Ormerod).

Figura 6 - Desvio Médio Absoluto (MAD) (A) e Raiz Quadrática do Erro Quadrático médio (RMSE) (B) para as árvores amostrais calculadas considerando estimativas de diâmetro pela metodologia proposta (Sólidos de revolução) e pelos modelos de afilamento (Demaerschalk; Kozak 1988; Kozak e outros 1969; e Ormerod).

most sample trees compared to the diameter estimated by the taper models (Figures 6A and 6B).

Considering the stems of sample trees composed of three parts (base, middle, and top), according to different forms along with them (Figure 3), we also verified that the methodology proposed in this study was the most accurate for different parts (Table 3), beyond the stem as a whole (Figure 6A and 6B).

Table 3 - Number of sample trees by position on the stems (base, middle, and top) where the methodology or models were more accurate

Tabela 3 - Número de árvores amostrais por posição no tronco (base, meio e topo) onde a metodologia ou modelos foram mais precisos

	Mean Absolute Deviation (MAD)			Root Mean Square Error (RMSE)		
Metodology/ Models	Base	Midle	Top	Base	Midle	Top
Solid of revolution	14	12	10	15	14	10
Kozak et al. (1969)	3	7	3	1	6	3
Demaerschalk (1972)	1	3	4	0	2	5
Ormerod (1973)	2	1	2	3	0	2
Kozak (1988)	7	4	8	8	5	7
Total	27	27	27	27	27	27

4. DISCUSSION

According to Husch et al. (2003), in general terms, the stem can be divided into three parts whose forms resemble different geometric forms: frustum of neiloid at the bottom; frustum of a paraboloid or in few cases, frustum of cone or cylinder at the central part; and cone or paraboloid at the top. The observed diameters (Figure 1) show a slight transition between forms along the stems for *Eucalyptus* trees at 60 months of age such that the central part of the trees stems resembles a cone (Figure 2).

Generating solids of revolution for each tree (Equation 3) and associating the observed diameters to geometric forms (Equation 4) allowed us to obtain statistically different volume estimates (*p-value* < 0.05) than those based on a single formula for all the sections (Figure 4), owing to the correct specification of the formulas to determine the volumes of each part of the stems. Consequently, the proposed methodology can avoid biased volume estimates fitting the volumetric and taper equation processes.

The taper equations fitted well to observed data, similar to the other studies that fitted taper equations to *Eucalyptus* sp. in Brazil (Campos et al., 2014; Souza et al., 2018), but presented inaccurate estimates of the diameters at the base of the stems. Kozak`s model (1988) presented multicollinearity between explanatory variables as also reported by Perez et al. (1990) and Kozak (2004).

Predicting the diameter at the base of the tree stems is the greatest challenge of the taper equations, given the presence of geometric distortions in these portions (Souza et al., 2018), producing somewhat biased estimates (Kozak, 2004; Rojo et al., 2010). The results showed better estimates of diameters outside the bark at this part of the stems for most of the sample trees by the proposed methodology (Table 3) compared to the traditional taper models.

The methodology developed in this study is not precisely a variable-exponent taper model. However, it uses a continuous function to describe the form of the stems from the ground (base) to the top, in association with the concepts of the solids of revolutions. In addition, the methodology decreases the subjectivity in determining of the relative height at the inflection point (where the stem changes form), which is essential for fitting segmented and variable-exponent taper equations. However, between certain limits, an inexact determination has little effect on the predictive capacity of the variable-exponent taper models (Perez et al., 1990, Alves et al., 2019).

The data for the proposed methodology can be obtained using the usual procedures for fitting the volume or tapering equations, implying no additional costs in data collection. Data such as the diameter (*dbh*) of the trees measured in forest inventories are necessary as input variables in the Equation 19 and 20 to estimate the total or partial volume stocks.

To improve the estimates from the equations system presented in this study (Equation 18, 19 and 20), information at stand level (age, site class, crown class, trees per hectare, basal area, and other) from forest inventory procedures can be used as auxiliary or independent variables in these equations, similar to what was carried out in modeling taper equations (Muhairwe et al., 1994; Sharma and Zhang, 2004; Sharma and Parton, 2009).

In the taper models, the stem volumes can be estimated by integrating the function or numerical integration method (Rojo et al., 2010; Burkhart and Tomé, 2012). According to the proposed methodology, there is no single function to estimate the diameters along the stems, but a function to estimate the coefficients of forms (*r*) used in the expression to estimate the diameters (Equation 3). Once the diameters can be estimated along the tree stem using the proposed methodology, the volumes of each section of the stem and the entire stem can be estimated using Equation 5, 6, and 7.

5. CONCLUSION

The results obtained in this study show the potential of applying the proposed methodology compared to traditional methodologies used for this purpose. It allowed: 1) the identification of forms along the stems; 2) provided accurate estimates of stem volumes respecting the forms in different parts of the stems; and 3) provided better estimates of diameters along the stems when compared to taper models.

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AUTHOR CONTRIBUTIONS

Conceptualization: Carlos Pedro Boechat Soares, Helio Garcia Leite e Gilson Fernades da Siva; Results analysis and discussion: Carlos Pedro Boechat Soares, Helio Garcia Leite e Gilson Fernades da Siva; Writing, review, and editing: Carlos Pedro Boechat Soares, Helio Garcia Leite e Gilson Fernades da Siva.

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